QP Code: 23/PT/13/VIIIB

POST-GRADUATE COURSE

Term End Examination — June, 2023/December, 2023

MATHEMATICS

Paper-8B : GRAPH THEORY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define degree sequence of a graph with examples.

   b) How many vertices are there in a 4-regular graph with 10 edges ?

   c) What is the difference between a circuit and a cycle ?

   d) Define a complete bi-partite graph.

   e) Draw a graph which is Hamiltonian but not Eulerian.

   f) Define a binary tree with examples.

   g) Define an incidence matrix of an undirected graph.

2. a) Prove that a simple graph with $n(\geq 2)$ vertices must have at least two vertices of equal degree.

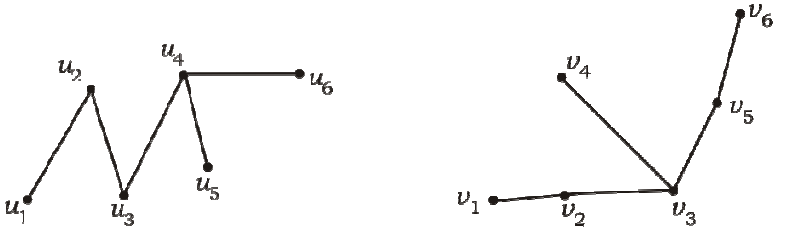
   b) Find the maximum number of edges in a simple graph with n vertices.

   c) Draw two different graphs with the same degree sequences $\{3, 3, 3, 3\}$. 3 + 3 + 4

3. a) Define isomorphisms of graphs.

   b) Prove that if the degree of each vertex of a graph is at least two then G contains a cycle.

   c) Show that the following graphs G_1 and G_2 are not isomorphic.



2 + 5 + 3

4. a) Prove that if a graph contains exactly two vertices of odd degree, then there exists a path between them.

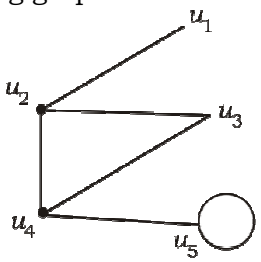
   b) Prove that a connected graph with n vertices has at least $n-1$ edges.

5 + 5

5. a) When is a complete bipartite graph $K_{m,n}$ Eulerian ? Justify your answer.

   b) Let G be a connected bipartite planar graph with n vertices and e edges. Prove that $n \leq 2e-4$. 5 + 5

6. a) Define an adjacency matrix of an undirected graph. Find adjacency matrix of the following graph.



   b) Define a rooted tree. Draw a binary tree with 4 internal vertices and 5 leaves. 5 + (2 + 3)

7. a) Let G be a simple connected graph with $n \geq 3$ vertices and m edges such that $m \geq \frac{1}{2}(n-1)(n-2)+2$. Show that G is Hamiltonian.

   b) Prove that a graph is a tree if and only if it is minimally connected.

5 + 5