

POST-GRADUATE COURSE
Term End Examination — June, 2023/December, 2023
MATHEMATICS

Paper-8A : DIFFERENTIAL GEOMETRY

Time : 2 hours]

[Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

(Notations and symbols have their usual meanings.)

Answer Question No. **1** and any *four* from the rest.

1. Answer any *five* questions : 2 × 5 = 10
- a) Show that $\delta_j^k B_k = B_j$.
- b) Prove that the Kronecker delta is a mixed tensor of type, to be determined by you.
- c) Evaluate

$$\left(g_{hj} g_{ik} - g_{hk} g_{ij} \right) g^{hj}$$
for a Riemannian Space of dimension n .
- d) Compute [11, 2] for cylindrical co-ordinates.
- e) Define Helix curve.
- f) When are two surfaces said to be isometric ?
- g) Define Mean Curvature of a surface.
2. a) If f is a scalar function of co-ordinates (x^1, x^2, \dots, x^n) , define grad f .
Show that grad f is a covariant vector.
- b) If $g_{ij} = 0$ for $i \neq j$
 $\neq 0$ for $i = j$,
show that $g^{ii} = \frac{1}{g_{ii}}$ (no summation) 5 + 5

3. a) If the metric is given by

$$ds^2 = (dx^1)^2 + 2(dx^2)^2 + 3(dx^3)^2 - 2dx^1 dx^3 + 4dx^2 dx^3,$$
 evaluate g and g^{ij} .
- b) Define the curl of a vector A_i . Evaluate $\text{Curl } A_i$. 5 + 5
4. a) Prove that the Riemann Curvature tensor of 2nd kind satisfies cyclic property.
- b) Show that the intrinsic derivative of an invariant coincides with its total derivative. 5 + 5
5. a) Find the differential equations of the geodesic for the line-element

$$ds^2 = (du)^2 + (\sin u)^2 (dv)^2.$$
- b) Calculate the Gaussian Curvature for the surface

$$ds^2 = (du)^2 + \mu^2 (dv)^2.$$
 5 + 5
6. a) Calculate the second fundamental form of the surface

$$r = (u, v, u^2 - v^2).$$
- b) Prove that the normal curvatures in the direction of the co-ordinate curves of the surface are b_{11}/a_{11} and b_{22}/a_{22} . 6 + 4
7. a) Deduce Codazzi Equations of a surface.
- b) Find the mean curvature of the surface

$$x^1 x^2 = x^3.$$
 6 + 4
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