POST-GRADUATE COURSE

Term End Examination — June, 2023/December, 2023

MATHEMATICS

Paper-5B : ELEMENTS OF CONTINUUM MECHANICS & SPECIAL THEORY OF RELATIVITY

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Prove that

   $$\varepsilon_{ijk}\,\varepsilon_{pqk} = \delta_{ip}\,\delta_{jq} - \delta_{iq}\,\delta_{jp}$$

   (symbols have their usual meanings)

   b) Show that all principal strains are real.

   c) Obtain constitutive equation of a perfect fluid.

   d) What do you understand by streamline of a particle ? Obtain the differential equation for it at a given instant *t*.

   e) A rod has a length of 100 cm. When it is moving with a speed of 0·80c along the direction of its length, find the length of the rod with respect to an observer at rest.

   f) For the velocity field given by $v_1 = kx_3$, $v_2 = kx_3$, $v_3 = k(x_1 + x_2)$, show that the motion is irrotational.

   g) Prove that on contraction from a tensor we get a new tensor whose order is less by two than that of the original tensor.

2. a) Given the following stress distribution

   $$(\tau_{ij}) = \begin{pmatrix} x_2 & -x_3 & 0 \\ -x_3 & 0 & -x_2 \\ 0 & -x_2 & T \end{pmatrix}$$

   Find T such that stress distribution is in equilibrium with the body force $\vec{F} = -g\,\vec{e}_3$.

   b) State and prove Kelvin's minimum energy theorem. 5 + 5

3. If Θ and θ be the angles between the line elements of length dL, δL and dl, δl respectively before and after deformation, then show that

$$\cos\theta - \cos\Theta \frac{dL}{dl}\frac{\delta L}{\delta l} = 2\eta_{ij} n_i m_j$$

where n_i and m_j are direction cosines of the line elements dl and δl respectively and η_{ij} are Eulerian finite strain tensor. 10

4. For relativity theory find the expressions for momentum and force, and find the relation between them. Show that in the low velocity the usual expression for Kinetic energy can be obtained from the above relativistic expression. 10

5. a) Establish Navier-Stokes equation of motion for incompressible viscous fluid. 5

   b) Find the principal directions of strain and corresponding direction ratios of principal strains for the following strain tensor

$$(e_{ij}) = \begin{pmatrix} a & b & 0 \\ b & -a & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$ 5

6. Prove that the equation of continuity is equivalent in both Lagrangian and Eulerian method. 10

7. a) Define the generalized Hooke's law for a linear elastic material. Obtain the stress-strain relation for an isotropic elastic body. 8

   b) Define Young's modulus. 2