**QP Code: 23/PT/13/VA**

# POST-GRADUATE COURSE

## Term End Examination — June, 2023/December, 2023

## MATHEMATICS

### Paper-5A : PRINCIPLES OF MECHANICS

Time : 2 hours ] [ Full Marks : 50

Weightage of Marks : 80%

**Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.**

***Use of scientific calculator is strictly prohibited.***

( *All Symbols have their usual meanings.* )

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Write the generalized coordinates for the following systems :

      (i) Simple pendulum

      (ii) Particle on the surface of a sphere

   b) Find the Lagrange's equation of motion using the Lagrangian

      $L = \frac{1}{2} q^2 \dot{q}^2 - q^3$.

   c) Obtain the Hamiltonian for a simple pendulum.

   d) Prove that the generalized momentum conjugate to a cyclic coordinate is a constant of motion of the system.

   e) For what value of the constant $\alpha$, the transformation $Q = \sqrt{2q}\, e^{-1+2\alpha} \cos p$, $P = \sqrt{2q}\, e^{-\alpha-1} \sin p$ will be canonical ?

   f) If $H$ is the Hamiltonian and $f$ is any function depending on position, momenta and time, show that

      $\frac{df}{dt} = \frac{\partial f}{\partial t} + \{ f, H \}$.

   g) State the brachistochrone problem. What does the brachistochrone represent ?

2. a) What are constraints ? What type of difficulties arises due to the constraints in the solution of mechanical problems and how are these removed ? Giving suitable example distinguish between scleronomic and rheonomic constraints. 7

   b) Explain the concepts of virtual displacement and virtual work. State the principle of virtual work. 3

3. a) What do you mean by homogeneity and isotropy of space and homogeneity of time ? Mention the conservation laws that follows from each. 4

   b) A bead slides on a wire in the shape of a cycloid described by equations

   $x = a(\theta - \sin\theta)$,

   $y = a(1 + \cos\theta)$,

   where $0 \le \theta \le 2\pi$.

   Find (i) the Lagrangian function, and (ii) the equation of motion.

   (The friction between the bead and the wire may be assumed negligible. ) 6

4. a) The Lagrangian for a dynamical system is

   $L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$.

   Identify the cyclic coordinates and the corresponding conservation law for the problem. 3

   b) Describe the Liouville's class of Lagrangians. What are the advantages of the problems of Liouville's type ? Obtain the transformed Lagrange's equations for such type of problems. 5

   c) Show that the dynamical system, for which $2T = r_1 r_2 (\dot{r}_1^2 + \dot{r}_2^2)$ and $V = \frac{1}{r_1} + \frac{1}{r_2}$, can be expressed as one of Liouville's types. 2

5. a) What is Hamiltonian function ? Derive Hamilton's equations of motion for a system of particles. 6

   b) Obtain the Hamilton's equations of motion for a plane pendulum and hence solve it. 4

6. a) If $\vec{r}$, $\vec{p}$ and $\vec{l}$ be the position vector, generalized momentum vector and angular momentum vector respectively and $\vec{n}$ be an arbitrary constant vector, then prove that

   (i) $\{\vec{r}, \vec{l} \cdot \vec{n}\} = \vec{n} \times \vec{r}$

   (ii) $\{\vec{r} \cdot \vec{p}, \vec{l} \cdot \vec{n}\} = 0$ 5

b) Explain invariance transformation. Prove that associated with an infinitesimal invariance transformation there is a constant of motion. 5

7. a) State Hamilton's principle and derive Lagrange's equations of motion from it. 4

b) Prove that the transformation

$$P = q \cot p \text{ and } Q = \log \frac{\sin p}{q}$$

is canonical. Obtain the generating function for the transformation. 6