POST-GRADUATE COURSE

Term End Examination — June, 2023/December, 2023

MATHEMATICS

Paper-4A : NUMERICAL ANALYSIS

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) How to calculate $\sqrt{6931} - \sqrt{6930}$ to avoid instability of calculation.

   b) Use appropriate formula to compute roots of $x^2 - 100 \cdot 001x + 1 = 0$.

   c) If a matrix A is factorised as a product of a lower triangular and an upper triangular matrix then how will you solve the system of linear equation $AX = b$?

   d) What is the difference between Crout's decomposition and Doolittle's decomposition of a square matrix ?

   e) What is shifted power method ?

   f) For $a > 0$, the convergent sequence $\{x_n\}$ where $x_{n+1} = \dfrac{x_n(x_n^2 + 3a)}{3x_n^2 + a}$ converges to l. What is the possible value of l ?

   g) What is the difference between single step and multistep methods to solve $\dfrac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$?

2. Write down canonical form of Poisson equation in $2D$, satisfying Dirichlet's type of boundary condition. Outline a scheme to solve it by finite difference method. 10

3. Deduce Adams-Moulten method to solve first order ODE with initial condition. Is it a implicit method ? Discuss. Outline the scheme to use it as a Predictor-Correcter method with a suitable method. 10

4. Write down general Gaussian quadrature formula with (n + 1) nodes in [– 1, 1]. Prove that its degree of precision cannot exceed ($2n$ + 1). How can it be used to find integral of the type $\int_a^b f(t)\,dt$. 10

5. a) Find the co-efficients that make the quadrature formula $\int_0^3 f(x)\,dx = c_1 f(-1) + c_2 f(0) + c_3 f(2) + c_4 f(4)$ such that it is exact for cubic polynomial.

   b) Outline the scheme of Romberg integration. 5 + 5

6. a) Prove that a small perturbation in the constant vector b of the system of linear equation $AX = b$ for $\det(A) \neq 0$ leads to small perturbation in the solution X if Card (A) is small.

   b) If α is a multiple root of $f(x) = 0$ of multiplicity r then prove that iteration scheme $x_{n+1} = x_n - \frac{rf(x_n)}{f'(x_n)}$ converges to α and its order of convergent is 2. 5 + 5

7. Find the solution of the system of linear equations :

$$2x_1 + 2x_2 + x_3 = 3$$
$$4x_1 + 3x_2 + 2x_3 = 5$$
$$3x_1 + 4x_2 + 2x_3 = 5$$

by Crout's reduction method. 10