

QP Code : 23/PT/11/XVIIB

POST-GRADUATE COURSE
Term End Examination — June, 2023/December, 2023
ECONOMICS
Paper-XVIIB : ADVANCED STATISTICS

Time : 2 hours]

[Full Marks : 50
Weightage of Marks : 80%

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Use of scientific calculator is strictly prohibited.

1. Answer any *four* of the following questions : $2\frac{1}{2} \times 4 = 10$

a) Is the following a probability density function ?

$$f(x) = \begin{cases} 2x, & 0 < x \leq 1 \\ 4 - 2x, & 1 < x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

b) A random variable has the following probability distribution :

X	4	5	6	8
Probability	0.1	0.3	0.4	0.2

Find the expectation and standard deviation of the random variable.

$$1 + 1\frac{1}{2}$$

c) What is the 'Central Limit Theorem' ?

d) Distinguish between 'Parameter' and 'Statistic'.

e) Differentiate between Type-I and Type-II errors.

f) What is a sufficient estimator ? Why is it said so ?

2. Answer any *four* of the following questions : $5 \times 4 = 20$

a) Three identical boxes I, II, III contains respectively 4 white and 3 red balls, 3 white and 7 red balls and 2 white and 3 red balls. A ball is chosen at random from a selected box and is drawn out of it. If the ball is found to be white, what is the probability that box II is selected ?

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- b) The manufacturer of a certain electronic component knows that 3% of his product is defective. He sells the component in boxes of 100 and guarantees that not more than 3% in any box will be defective. What is the probability that a box will fail to meet the guarantee ?
[Given $e^3 = 20.1$]
- c) If T_1, T_2, T_3 are independent unbiased estimates of θ and all have the same variance, which of the following unbiased estimates of θ would you prefer ?
- $\frac{T_1 + 2T_2 + T_3}{4}$
 - $\frac{3T_1 + T_2 - 2T_3}{2}$
 - $\frac{T_1 + T_2 + T_3}{3}$
- d) A random sample of size 20 from a normal population gives a sample mean of 42 and standard deviation of 6. Test the hypothesis that the population mean is 44. State clearly the alternative hypothesis you allow for and the level of significance adopted. [Note the table value of $t_{0.025} = 2.09$ and $t_{0.005} = 2.86$ respectively for t distribution with 19 degrees of freedom]
- e) Derive the formulae for standard error of sample mean in case of simple random sample with replacement and simple random sample without replacement in case of finite population.
- f) A random sample of 100 items taken from a larger batch of articles contain 5% defective items :
- Set up 95% confidence interval for the proportion of defective items in a batch.
 - The batch contains 2697 items. Set up 95% confidence limit for the proportion of defective items. $2\frac{1}{2} + 2\frac{1}{2}$

3. Answer any *two* of the following questions : 10 × 2 = 20

a) i) There is a 50-50 chance that a contractor firm *A* will bid for construction of a shopping mall. Another firm *B* submits a bid and the probability is $\frac{3}{4}$ that it will get the job, provided that firm *A* does not bid. If firm *A* submits a bid, the probability of firm *B* will get the job is only $\frac{1}{3}$, what is the probability that firm *B* will get the job ? 5

ii) A person takes a step forward with the probability 0.25 and backward with probability 0.75. What is the probability that at the end of 7 steps he will be one step away from the starting point ? 5

b) i) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and S.D. Rs. 70 and Rs. 5, respectively, find the expected number of workers whose weekly wages are :

I. in between Rs. 66 and Rs. 72.

II. less than Rs. 66

III. more than Rs. 72

Given that $\int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 0.1554$ and 0.2881 , accordingly as

$z = 0.4$ and 0.8 . 2 + 2 + 2

ii) Distinguish between :

I. Critical region and acceptance region

II. Null hypothesis and alternative hypothesis. 2 + 2

c) i) Show that sample mean based on a simple random sample with replacement (SRSWR) is an unbiased estimator of the population mean. 5

- ii) Write short notes on the following :
- I. Maximum Likelihood Estimation (M.L.E.)
 - II. Properties of Normal Distribution. $2\frac{1}{2} + 2\frac{1}{2}$
- d) i) How would you distinguish between Standard Error (SE) and Standard Deviation (SD) ?
- ii) Explain 'Stratified Sampling'.
- iii) In a big city 325 men out of 600 men were found to be cricket lovers. Does this information support the conclusion that the majority of men in the city love cricket ? (State your hypothesis clearly)
- [Note that the tail area of standard normal curve for $Z \geq 1.645$ is 5%] 2 + 3 + 5
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